ext. . to an f-invariant fibration $\tilde{W}^{uu}$ over a neighbourhood of V in W . If $x \in N^s(p)$ define $N^u(x) = x + N^u(p) \subset N(p)$; then near V we construct diffeomorphisms $g(x) : N^u(x) \to \tilde{W}^{uu}(h_s x)$ such that $g(x)^{-1}.f.g(x)$ is C^1-close to Nf on $N^u(x)$ (for example by using a projection of $N^u(x)$ onto $\exp_p^{-1}\tilde{W}^{uu}(h_s x)$) and then $\{g(x) | x \in N^s(p)\}$ defines g from a neighbourhood of V in N to a neighbourhood of V in M such that $g^{-1}fg$ is conjugate (by the extended Hartman's theorem) to Nf. Hence f is conjugate to Nf.

COROLLARY. If the restriction of f to V is structurally stable, then f is structurally stable in a neighbourhood of V.

(22) Topologically transitive diffeomorphisms of T^4 — M. Shub

Let $f:M \to M$ be a diffeomorphism (M compact, connected, C^∞) with p a periodic point of f of period n. The following is easy to prove:

THEOREM 1. Let $W^s(p)$ ($W^u(p)$) be the "stable (unstable) manifold" tangent to the subspace of TM(p) corresponding to eigenvalues of $Df^n(p)$ with modulus <1 (>1) (f need not be hyperbolic). If $W^s(p)$ and $W^u(p)$ are dense in M then f is topologically transitive.

Question: Let A be an ergodic automorphism of T^n. Is a $C^1(C^2)$ perturbation of A topologically transitive?

Let E be a compact C^∞ manifold ($\partial E = \emptyset$), Λ a topological space. A locally trivial fibration $\pi:E \to \Lambda$ is a C^r-regular fibration if $\pi^{-1}(\lambda)$ is a C^r-submanifold of E ($\lambda \in \Lambda$) and the map $x \mapsto T_x\pi^{-1}(\pi x)$ is continuous. A perturbation of π is a homeomorphism $h:E \to E$, C^r on fibres, such that πh^{-1} is a C^r-regular fibration, h is C^0-close to the identity, and Dh along fibres is C^1-close to the identity. We say π is a C^r-equivariant fibration for a diffeomorphism $F:E \to E$ and homeomorphism $f:\Lambda \to \Lambda$ if $\pi F = f\pi$.

THEOREM 2. Let $f:M \to M$ be an Anosov diffeomorphism with periodic points dense in M, $f(m_0) = m_0$, $\pi:E \to M$ a C^r-equivariant fibration for $F:E \to E$ and f such that $F|\pi^{-1}(m_0)$ is Anosov with periodic points dense. Then F is topologically transitive.

(The proof uses standard stable manifold theory and the fact that $W^s(\pi^{-1}(m_0)) = \pi^{-1}(W^s(m_0))$.)

THEOREM 3. (Equivariant fibration theorem). Let $\pi:E \to M$ be a C^r-equivariant fibration for F, f where f is Anosov. If f is "more hyperbolic than F" along the fibres then for any sufficiently small C^1-perturbation G of F there is a perturbation π' of π such that π' is a C^1-equivariant fibration for G, f.

Using this we obtain

THEOREM 4. Let π, F, f be as in Theorems 2, 3. Then any sufficiently small C^1 perturbation of F is topologically transitive.

As a special case we have the example on T^4 as described in (16) (page 28).

These two lectures represent part of joint work with Hirsch and Pugh, which will appear.

(23) Ω-explosions

J. Palis

A diffeomorphism f of a manifold M (M compact) satisfies Axiom A if the non-wandering set $\Omega = \Omega(f)$ has a hyperbolic structure, and $\overline{\mathrm{Per}\,(f)} = \Omega$. There is then a 'spectral decomposition' of Ω into components Ω_i on each of which f is topologically transitive (see [3]). An n-cycle on Ω is a sequence $\Omega_0, \Omega_1, \ldots, \Omega_{n+1}$ with $W^s(\Omega_i) \cap W^u(\Omega_{i+1}) \neq \emptyset$, $\Omega_{n+1} = \Omega_0$ and otherwise $\Omega_i \neq \Omega_j$ for $i \neq j$.

THEOREM [2]. If f satisfies axiom A and there is an n-cycle on