to an $f$-invariant fibration $\tilde{\mathcal{M}}^u$ over a neighbourhood of $V$ in $\mathcal{W}$. If $x \in N^u(p)$ define $N^u(x) = x + N^u(p) \subset N(p)$; then near $V$ we construct diffeomorphisms $g(x) : N^u(x) \to \tilde{\mathcal{M}}^u(h_s x)$ such that $g(x)^{-1} f g(x)$ is $C^1$-close to $N_f$ on $N^u(x)$ (for example by using a projection of $N^u(x)$ onto $\exp_p^{-1} N^u(h_s x)$) and then $(g(x)|x \in N^s(p))$ defines $g$ from a neighbourhood of $V$ in $\mathcal{N}$ to a neighbourhood of $V$ in $\mathcal{M}$ such that $g^{-1} f g$ is conjugate (by the extended Hartman's theorem) to $N_f$. Hence $f$ is conjugate to $N_f$.

**Corollary.** If the restriction of $f$ to $V$ is structurally stable, then $f$ is structurally stable in a neighbourhood of $V$.

(22) **Topologically transitive diffeomorphisms of** $T^h$  

M. Shub

Let $f : M \to M$ be a diffeomorphism ($M$ compact, connected, $C^\infty$) with $p$ a periodic point of $f$ of period $n$. The following is easy to prove:

**Theorem 1.** Let $W^s(p)$ ($W^u(p)$) be the "stable (unstable) manifold" tangent to the subspace of $TM(p)$ corresponding to eigenvalues of $DF^s(p)$ with modulus $< 1$ ($> 1$) (if $f$ need not be hyperbolic). If $W^s(p)$ and $W^u(p)$ are dense in $M$ then $f$ is topologically transitive.

**Question:** Let $A$ be an ergodic automorphism of $T^R$. Is a $C^1$($C^2$) perturbation of $A$ topologically transitive?

Let $E$ be a compact $C^\infty$ manifold ($\partial E = \emptyset$), $A$ a topological space. A locally trivial fibration $\pi : E \to A$ is a $C^r$-regular fibration if $\pi^{-1}(\lambda)$ is a $C^r$-submanifold of $E$ ($\lambda \in A$) and the map $x \mapsto T_x \pi^{-1}(\pi x)$ is continuous. A perturbation of $\pi$ is a homeomorphism $h : E \to E$, $C^r$ on fibres, such that $wh^{-1}$ is a $C^r$-regular fibration, $h$ is $C^0$-close to the identity, and $Dh$ along fibres is $C^1$-close to the
identity. We say \( \tau \) is a \( C^r \)-equivariant fibration for a diffeomorphism \( F: E \to E \) and homeomorphism \( \varphi : A \to A \) if \( \tau F = \tau \varphi \).

**Theorem 2.** Let \( \tau : M \to M \) be an Anosov diffeomorphism with periodic points dense in \( M \), \( \tau(m_0) = m_0 \). Let \( \pi : E \to M \) be a \( C^r \)-equivariant fibration for \( F: E \to E \) and \( \tau \) such that \( \pi^{-1}(m_0) \) is Anosov with periodic points dense. Then \( F \) is topologically transitive.

(The proof uses standard stable manifold theory and the fact that \( \pi^{-1}(w^s(m_0)) = \pi^{-1}(w^s(m_0)) \).

**Theorem 3.** (Equivariant fibration theorem). Let \( \pi : E \to M \) be a \( C^r \)-equivariant fibration for \( F \), \( \tau \), where \( \tau \) is Anosov. If \( \tau \) is "more hyperbolic than \( F \)" along the fibres then for any sufficiently small \( C^1 \)-perturbation \( G \) of \( F \) there is a perturbation \( \tau' \) of \( \tau \) such that \( \tau' \) is a \( C^1 \)-equivariant fibration for \( G \), \( \tau \).

Using this we obtain

**Theorem 4.** Let \( \tau \), \( F \), \( \tau \) be as in Theorems 2, 3. Then any sufficiently small \( C^1 \)-perturbation of \( F \) is topologically transitive.

As a special case we have the example on \( T^1 \) as described in (16) (page 28).

These two lectures represent part of joint work with Hirsch and Pugh, which will appear.

(23) \( \Omega \)-explosions

A diffeomorphism \( f \) of a manifold \( M \) (\( M \) compact) satisfies Axiom A if the non-wandering set \( \Omega = \Omega(f) \) has a hyperbolic structure, and \( \text{Per}(f) = \Omega \). There is then a 'spectral decomposition' of \( \Omega \) into components \( \Omega_i \) on each of which \( f \) is topologically transitive (see [3]). An \( n \)-cycle on \( \Omega \) is a sequence \( \Omega_0, \Omega_1, \ldots, \Omega_{n+1} \) with \( \omega^u(\Omega_i) \cap \omega^s(\Omega_{i+1}) \neq \emptyset \), \( \Omega_{n+1} = \Omega_0 \) and otherwise \( \Omega_i \neq \Omega_j \) for \( i \neq j \).

**Theorem 5.** If \( f \) satisfies axiom A and there is an \( n \)-cycle on