extend enough of the analysis to give a proof of the theorem in
the case of the principal bundle.

REFERENCES:

Some smooth ergodic systems, Russian Mathematical Surveys 22
(1967) 103-167.

Statistik der Lösungen geodätischer
Problems von unstabilen Typus, II,

(16) Instability

M. Shub

Diffeomorphisms \( f \colon X \to X \), \( g \colon Y \to Y \) are topologically
conjugate if there is a homeomorphism \( h : X \to Y \) such that \( hf = gh \).
Let \( M \) be a \( C^r \) manifold with \( \partial M = \emptyset \), and \( \text{Diff}^r(M) \) the space of
\( C^r \) diffeomorphisms of \( M \), with the \( C^r \) topology: then \( f \in \text{Diff}^r(M) \)
is structurally stable if there is a neighbourhood \( U_f \) of \( f \) in
\( \text{Diff}^r(M) \) such that \( g \in U_f \implies f,g \) topologically conjugate. It
was conjectured but proved to be false [2] that structurally
stable diffeomorphisms were always dense in \( \text{Diff}^r(M) \).

Define \( \Omega(f) = \{ x \mid \text{given open } U \ni x \text{ there exists } m > 0 \)
with \( r^m(U) \cap U \neq \emptyset \} \). Then \( f \in \text{Diff}^r(M) \) is \( \Omega \)-stable if \( \exists U_f \)
with \( g \in U_f \implies f|\Omega(f), g|\Omega(g) \) topologically conjugate. An
example due to Abraham and Smale in 1967 [1] showed that \( \Omega \)-stable
functions also fail to form an open dense set in \( \text{Diff}^r(M) \).
However, we may define a weaker property: \( f \in \text{Diff}^r(M) \) is
topologically \( \Omega \)-stable if \( \exists U_f \) with \( g \in U_f \implies \Omega(f) \) homeomorphic
to \( \Omega(g) \).

Problem: Do topologically \( \Omega \)-stable diffeomorphisms form an open
dense set in \( \text{Diff}^r(M) \)?
The following shows that topological $\Omega$-stability is weaker than $\Omega$-stability:

**Theorem.** There exists an open set $U \subseteq \text{Diff } (T^2 \times T^2)$ with $g \in U \Rightarrow g$ topologically $\Omega$-stable but not $\Omega$-stable.

The set $U$ is constructed using an Anosov diffeomorphism $A : T^2 \to T^2$ and a related 'deformed' diffeomorphism $DA$ defined by splitting a saddle of $A$ into a sink and two saddles by a large $C^1$-isotopy. A diffeomorphism $G : T^2 \times T^2 \to T^2 \times T^2$ is defined by $G(x,y) = (A^2x, \phi(x)y)$ where $\phi : T^2 \to \text{Diff}(T^2)$ is constructed so that $\phi(x) = A$ outside a small disc of $T^2$, $\phi(x) = DA$ inside a smaller disc, and $\phi$ gives the isotopy $A = DA$ in between. Then $G(g) = T^2 \times T^2$, which is $G(G')$ for all $G'$ close enough to $G$ by the "equivariant fibration theorem" (p. 40). But $G$ is not structurally stable and so not $\Omega$-stable.

**REFERENCES:**


[2] Smale, S.

(To appear in Proceedings of AMS Summer Institute on Global Analysis, Berkeley 1969)


(17) One-parameter families of diffeomorphisms

We give some results concerning the generic behaviour of periodic points of $C^r$ maps ($1 < r < \infty$) $f : P \times M \to M$, where $P$ and $M$ are $C^r$ manifolds, $\dim P = 1$, and the map $F_{\mu} : M \times M$ given by $f_{\mu}(x) = f(\mu, x)$ is a diffeomorphism for each $\mu \in P$. For given $P, M$ we denote by $\mathcal{F}$ the set of all such $f$'s with the Whitney $C^r$ topology.

Similar problems have been studied by J. Sotomayor (two-dimensional flows) in [2] and K. Meyer (two-dimensional