extend enough of the analysis to give a proof of the theorem in the case of the principal bundle.

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(16) Instability

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Diffeomorphisms $f : X \to X$, $g : Y \to Y$ are topologically conjugate if there is a homeomorphism $h : X \to Y$ such that $hf = gh$. Let M be a C^∞ manifold with $\partial M = \emptyset$, and $\mathrm{Diff}^r(M)$ the space of C^r diffeomorphisms of M, with the C^r topology: then $f \in \mathrm{Diff}^r(M)$ is structurally stable if there is a neighbourhood U_f of f in $\mathrm{Diff}^r(M)$ such that $g \in U_f \Longrightarrow f,g$ topologically conjugate. It was conjectured but proved to be false [2] that structurally stable diffeomorphisms were always dense in $\mathrm{Diff}^r(M)$.

Define $\Omega(f) = \{x \mid$ given open $U \ni x$ there exists $m > 0$ with $f^m(U) \cap U \neq \emptyset\}$. Then $f \in \mathrm{Diff}^r(M)$ is Ω-stable if $\exists\ U_f$ with $g \in U_f \Longrightarrow f|\Omega(f), g|\Omega(g)$ topologically conjugate. An example due to Abraham and Smale in 1967 [1] showed that Ω-stable functions also fail to form an open dense set in $\mathrm{Diff}^r(M)$. However, we may define a weaker property: $f \in \mathrm{Diff}^r(M)$ is topologically Ω-stable if $\exists\ U_f$ with $g \in U_f \Rightarrow \Omega(f)$ homeomorphic to $\Omega(g)$.

Problem: Do topologically Ω-stable diffeomorphisms form an open dense set in $\mathrm{Diff}^r(M)$?

The following shows that toplogical Ωstability is weaker than Ω-stability:

THEOREM. There exists an open set $U \subset \mathrm{Diff}(T^2 \times T^2)$ with $g \in U \Longrightarrow g$ topologically Ω-stable but not Ω-stable.

The set U is constructed using an Anosov diffeomorphism $A : T^2 \to T^2$ and a related 'deformed' diffeomorphism DA defined by splitting a saddle of A into a sink and two saddles by a large C^1-isotopy. A diffeomorphism $G : T^2 \times T^2 \to T^2 \times T^2$ is defined by $G(x,y) = (A^2x, \phi(x)y)$ where $\phi : T^2 \to \mathrm{Diff}(T^2)$ is constructed so that $\phi(x) = A$ outside a small disc of T^2, $\phi(x) = DA$ inside a smaller disc, and ϕ gives the isotopy $A \simeq DA$ in between. Then $\Omega(G) = T^2 \times T^2$, which is $\Omega(G')$ for all G' close enough to G by the "equivariant fibration theorem" (p. 40). But G is not structurally stable and so not Ω-stable.

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(17) One-parameter families of diffeomorphisms

P.Brunovksý

We give some results concerning the generic behaviour of periodic points of C^r maps $(1 < r < \infty)$ $f : P \times M \to M$, where P and M are C^r manifolds, dim P = 1, and the map $F_\mu : M \to M$ given by $f_\mu(x) = f(\mu, x)$ is a diffeomorphism for each $\mu \in P$. For given P,M we denote by $\mathfrak{F}$ the set of all such f's with the Whitney C^r topology.

Similar problems have been studied by J. Sotomayor (two-dimensional flows) in [2] and K. Meyer (two-dimensional