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DEFORMING FACTORS IN IDIOPATHIC SCOLIOSIS: A MATHEMATICAL TOOL

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In the quest for an etiology of idiopathic scoliosis, extensive research activity is conducted in many fields presented in this issue. In most of these fields the connection between a systemic phenomenon and a local malformation has yet to be made. In spite of the fact that spinal deformity may be the expression of a variety of pathological origins, we hypothesize that the initiation and progression can be addressed mathematically. This aim of this work is to develop a mathematical model associating geometric data with local deforming factors, which, in turn, can be correlated with pathological entities that have not yet been discerned by current diagnostic practice or considered as initiating factors in scoliosis.

We hypothesize that there exist one or more local factors leading to an initiation and/or progression of spinal deformity. This work is aimed in locating these factors and predicting their severity so that idiopathic scoliosis can be treated early. Our strategy to test our hypothesis by means of computer simulations is based on a mathematical model of the spine: local anatomical deforming factors are entered as inputs to a computer program and outputs of runs are compared with scoliotic deformities. Ambiguous results leading to the null hypothesis may indicate a non-musculoskeletal etiology or too many local deforming factors to be of use.

This concept is not new and had been related to in the past by a number of authors. Numerous attempts to model the spine can be found in the literature,[2–6,8–19] yet we are not aware of any model that is currently clinically useful to a specific patient, let alone that identifies initiating and deforming factors. Models based on kinematics or finite element methods involve assumptions about forces on the spine as well as material properties of living tissues, which resist quantification. In vivo measurements of mechanical properties and forces are plagued with inaccuracies akin to Heisenberg's uncertainty principle: namely, measurement changes what is being measured. Furthermore, changes leading to scoliotic spines may be smaller by orders of magnitude than that captured by the finite element method. Finite element analysis is used to solve numerically the nonlinear partial differential equations arising in the theory of elasticity if indeed that theory is even relevant to the problem.

The model we developed has the virtue that it relies only on data absolutely measurable and reproducible. For the spine, these are geometric quantities such as exact coordinates of vertebrae in space and limits on ranges of motion. We test our hypothesis by means of computer simulations: local anatomical deforming factors are entered as inputs to a computer program and outputs of runs are compared with scoliotic deformities. These simulations are based on algorithms derived from a mathematical concept of the spine that is described later.

The fact that our model requires accurate three-dimensional geometric data of the spine introduces the problem of data acquisition. Ideally, we would like 1-mm CT cuts of the spine of a standing patient, which is not feasible. In this work we also address alternative methods of obtaining suitable data. The current status of the project is that a tool has been developed and is about to be tested on real anatomical data. The protocol for data acquisition is not finalized yet.

MATHEMATICAL MODEL

We model the spine with a function, which for reasons described below we call spinal energy. This function expresses nature's intent to balance the head level over the pelvis while maintaining as much flexibility as possible. These two desiderata conflict, and a compromise is achieved by supposing that the spine obeys the following principle: spinal shape minimizes spinal energy. Spinal energy is a quadratic function of 275 displacements that measures deviations from the above desiderata and depends on 216 variables.

Displacements are of two types: distance and orientation differences. For example, an offset of a vertebra from the vertical is a distance difference, while a rotation of one vertebra with respect to a neighbor is an orientation one. The number 275 results from the fact there are 25 spaces (23 intervertebral spaces plus L5–S1 and C0–C1) and we use two coordinates to describe a distance and nine coordinates to describe an orientation difference [$25 \times 2 + 25 \times 9 = 275$]. Variables are components of orthogonal frames that specify three-dimensional (3-D) orientation of vertebrae. Each orthogonal frame is specified by nine numbers. The number 216 results from the fact that S1 and C0 have a fixed orientation while L5 to C1 are variable [$24 \times 9 = 216$].

The reason for choosing the term spinal energy is that it resembles energy as given by Hooke's law. The energy function of that law is one half the square of a displacement weighted by a stiffness coefficient. Spinal energy is a sum of squares of displacements, also weighted by quantities that we call stiffness coefficients. Furthermore, limits on ranges of motion of vertebrae are incorporated in the stiffness coefficients.

Spinal energy should not be confused with a real energy due to bodily movement. Rather, it is an ideal quantity that sums up the complex of muscular activities, metabolic processes, and growth forces, which drive the shaping of the erect spine.

Finding the minimum of function of many variables subject to constraints is a calculus problem. The Lagrange method of undetermined multipliers solves such a problem by setting the gradient of a function, which is restricted to a surface specifying the constraints, equal to 0. It leads to a set of nonlinear equations whose solution is found by using a Newton method.[1] Due to its size, the problem requires the computational power of modern computers.

Spinal energy depends on perhaps as many as 423 additional parameters. The additional parameters are divided into two types: universal ones, which are stiffness coefficients, and patient-specific ones. The stiffness coefficients are empirical quantities incorporating all of the unknown processes shaping the spine; they are universal in that they are supposed to be valid for a large class of spines but can be determined from a small sample from that class. This is done by solving a problem inverse to that of finding spinal configurations. We describe the inverse problem below. The patient-specific parameters are heights of vertebral bodies, thickness of discs, and limitations on ranges of vertebral motion—the only precise anatomical values that can be measured directly from imaging studies.

There are two ways of using the equations arising from setting the gradient of spinal energy equal to 0. On the one hand, the stiffness coefficients can be treated as known quantities and the variables specifying spinal configuration as unknowns. On the other hand, the spinal configurations, which can be obtained from radiological data, can be treated as knowns and stiffness coefficients as unknowns.

In order to test the validity of our model, we take a small sample from a certain class of spines, for example, defined by type of deformity, gender, growth status, etc. From this sample we determine the stiffness coefficients by solving the inverse problem. We then run the model using values for patient-specific parameters for other spines in the class and compare the simulated spines with the actual ones. Furthermore, comparing the difference between terms in the spinal energy function of a deformed spine with those from a normal class may localize a source of deformity.

The aforementioned Newton method is a dynamical systems approach to finding solutions to nonlinear equations. It is an iterative method that converges very rapidly provided the starting point is in the basin of attraction of a solution. (In the mathematical theory of dynamical systems, the concept of basin of attraction is the analogue of a black hole in astronomy.) Otherwise, iterations may produce inextricable results. The size of the basin of attraction for the minimum of spinal energy may be of significance with respect to the progression of scoliotic deformity. For example, our simulations have indicated that the basin of attraction for a normal spine is quite large. However, for a spine with a deformity leading to a large three-dimensional distortion, the basin of attraction is very small. In order to get into such a basin, we were forced to run a sequence of simulations gradually increasing the deformity. This is consistent with the fact that an abrupt spinal trauma does not lead to scoliosis, it being rather the result of a gradual process.

In addition to locating anatomical deforming factors of a scoliotic spine of a specific patient, simulations can be run with trial correcting factors as inputs, the outputs then compared to a normal spine. This might be useful in guiding surgical treatment.

With the idea of the mathematical tool in mind, two major avenues of activity are taken: (1) constructing the algorithm and program that can handle this immense amount of data generated by each patient; and (2) the major technical problem of data acquisition. Digital data from CT scans and digital (or scanned) long x-ray films are the inputs to a computer program. This program incorporates mathematics capable of getting three-dimensional reconstructions of spines in their full ranges of motion, tracking evolution of deformities.

Data Acquisition. Protocols for data acquisition using safe radiation dosages are being developed and tested in conjunction with this study. Decision making in the treatment of progressive spinal diseases is typically based on longitudinal studies, which involve the imaging and visualization of a patient's spine over a period of years. Surgery is deemed necessary if spinal deformation exceeds a certain degree of severity, and surgeons presently rely on two-dimensional (2-D) measurements obtained from x-rays to quantify the deformation. Working only with 2-D measurements seriously limits the surgeon's ability to infer three-dimensional (3-D) spinal pathology. It is difficult to conceive, let alone quantify vertebral rotations, curve translations, and other changes in orientations of spinal elements. Standard CT scanning and 3-D reconstruction is not a practical solution for obtaining 3-D spinal measurements of scoliotic patients. Scanning the spine requires a large number of axial slices, and therefore exposes a patient to high doses of radiation. In scoliosis the dosage problem is seriously compounded because of the need of repeated scans over a period of years. Longitudinal studies of development of curves, are a fundamental part in the quest for etiology.

In order to address this problem, we have developed two new digital imaging-based methods of 2-D spinal visualization that produce 3-D models of the spine by integrating a very small number of axial CT slices with data obtained from either CT scout images or 2-D digital x-rays.

In the first method the scout data are converted to sinogram data, and then processed by a tomographic image reconstruction algorithm.[7] The resulting slices are combined with the CT slices.

In the second method, an edge-detection algorithm is applied to find vertebral boundaries in the scout or digital image data. These edges are then used as linear constraints to determine 2-D convex hulls of vertebrae, and these 2-D contours are combined with vertebral contours extracted from CT slices to form a 3-D model.

With these methods we are able to: (1) image the complete spine in enough detail to quantify the 3-D nature of a deformity; and (2) deliver far less radiation to the patient than with standard CT scanning. This system allows the surgeon to choose the amount of detail to trade-off against radiation dosage.

CONCLUSION

We plan to incorporate raw data available from imaging scanners into a mathematical tool. The tool will be capable of constructing a good 3-D representation of the patient's spine, rendering the spine in neutral position and through the full normal and pathological range of motion. In addition, the tool will be capable of incorporating growth and 3-D progression of the deformity over time within the building blocks of the spine, i.e., the vertebrae and the spaces between them, and of reducing deformities on the screen. The latter function can indicate the existence of a primary deforming factor(s).

Success of this project will lead to the ability to identify various deforming factors and isolate groups with these factors within the idiopathic scoliosis patient population, for example, calculating the overgrowth of the anterior column, locating a growth asymmetry in the pedicles with initiation of rotation, local changes in disc stiffness, or other entities not yet discerned. The model will also enable prediction of deterioration by calculating the different local growth rates and the changes in ranges of motion and location in space, thus revealing information that may lead to the possibility of early arrest of deformity. Planning for the optimal surgical correction in an existing deformity will be facilitated by the option to insert small local changes in the geometry of the spine in order to obtain good balance. Finally, we emphasize that this project is based only on the use of strictly measurable geometric data.

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