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CORRECTION TO "HÖLDER FOLIATIONS"

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A. Török has pointed out to us the need for a better proof of [1, Theorem B]. Accordingly, the first two full paragraphs on [1, p. 539] should be replaced with the following argument.

We are trying to show that the subfoliation of the center unstable leaves by the strong unstable leaves is of class C^1. Let W denote the disjoint union of the center unstable leaves:

$$W = \bigsqcup W^{cu}(p).$$

It is a nonseparable manifold of class C^1. Partial hyperbolicity implies that its tangent bundle $TW = E^{cu}$ is continuous. The restriction of TM to W is a C^1 bundle $T_W M$ that contains the C^0 subbundle TW. Since f is a diffeomorphism of class C^2, the tangent map

$$Tf : T_W M \longrightarrow T_W M$$

is a C^1 bundle isomorphism. As in the proof of Theorem A (see [1, pp. 527–538]), approximate E^u, E^{cs} by smooth bundles $\widetilde{E}^u$, $\widetilde{E}^{cs}$, and express Tf with respect to the splitting $TM = \widetilde{E}^u \oplus \widetilde{E}^{cs}$ as

$$\begin{pmatrix} A & B \\ C & K \end{pmatrix}.$$

Let $\widetilde{\mathcal{P}}(1)$ be the bundle over W whose fiber at p is the set of linear maps $P : \widetilde{E}^u_p \to \widetilde{E}^{cs}_p$ such that $\|P\| \le 1$. The linear graph transform sends P to

$$\Gamma_{Tf}(P) = \big(C + KP\big) \circ \big(A + BP\big)^{-1}.$$

It is a bundle map that covers the identity on W, contracts fibers by approximately $\|K\|\|A^{-1}\| \doteq \|T^c f\|/m(T^u f)$, and contracts the base, at worst, by approximately $m(A) \doteq m(T^c f)$. The unique invariant section $p \mapsto P_p$ of $\widetilde{\mathcal{P}}(1)$ of Γ_{Tf} has graph $P_p = E^u_p$. Center bunching implies that

$$\big(\text{fiber contraction}\big)\big(\text{base contraction}\big)^{-1} \doteq \frac{\|T^c f\|}{m\big(T^u f\big)}\big(m\big(T^c f\big)\big)^{-1} < 1.$$

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So fiber contraction dominates base contraction, and the invariant section is of class C^1. That is, E^u is a C^1 bundle over the C^1 manifold W. Since E^u is tangent to the foliation $\mathcal{W}^u$, it is integrable.

Frobenius's theorem states that the foliation tangent to a C^k integrable subbundle of TW is of class C^k, in the sense that there is a C^k atlas of foliation charts covering the manifold W. Strictly speaking, the proof requires that the underlying manifold be of class C^{k+1}, so we need to recheck the result in the case of the C^1 manifold W.

Locally, $W^{cu}(p)$ is the graph of a C^1 function $g : E_p^{cu} \to E_p^s$. The linear projection $\pi : E_p^{cu} \times E_p^s \to E_p^{cu}$ restricts to a C^1 diffeomorphism $\pi_p : W^{cu}(p) \to E_p^{cu}$,

$$\pi_p : \big(x, g(x)\big) \longmapsto x.$$

The tangent to π gives a C^1 bundle surjection

$$T\pi : T_{W^{cu}(p)}M \longrightarrow T\big(E_p^{cu}\big).$$

The restriction of $T\pi$ to $E^u|_{W^{cu}(p)}$ agrees with $T\pi_p$, which implies that

$$T\pi_p : E^u|_{W^{cu}(p)} \longrightarrow T\pi_p\big(E^u|_{W^{cu}(p)}\big)$$

is a C^1 bundle isomorphism. The latter bundle is C^1 and is integrated by the foliation $\pi_p(\mathcal{W}^u)$. Since $E^{cu}(p)$ is smooth (being a plane), we can apply Frobenius's theorem to conclude that the foliation $\pi_p(\mathcal{W}^u)$ is C^1. Therefore, the foliation $\mathcal{W}^u|_{W^{cu}(p)}$ is also of class C^1.

References

[1] C. Pugh, M. Shub, and A. Wilkinson, *Hölder foliations*, Duke Math. J. **86** (1997), 517–546.

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