These are clearly related to generation theory of nonlinear semigroups, the dissipative realizations in $L^1$ and in $H^{-1}$ of $\Delta \circ \varphi$ for maximal monotone $\varphi$, and other well-known proofs by monotonicity and compactness methods. These interconnections are acknowledged but not developed, although this would have been interesting.

The highlight of the exposition is the set of Bibliographical Remarks which follow each chapter. These compose a remarkably exhaustive (and exhausting) account of the origin and credits for the many research works which are summarized or used. The weak point is the overwhelming background required of the reader. In addition to the usual linear functional analysis, measure and distribution theory, variational theory of elliptic PDE and Sobolev spaces as known to applied mathematical analysts and certain other scientists, the author cites facts from approximation theory, convex analysis, singular integral operators, interpolation of operators, degree theory, linear semigroups and certain topics for quasilinear elliptic or parabolic PDE. In spite of these assumptions on the readers' background, it occurs that one is referred elsewhere for the details of a proof, these often seeming to this technician to be the crux moves.

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*Information, Uncertainty, Complexity.* By J. F. TRAUB, G. W. WASILKOWSKI AND H. WOŹNIAKOWSKI. Addison-Wesley, Reading, MA, 1983. xii + 176 pp. $34.95. ISBN 0-201-07890-2.

*Information, Uncertainity, Complexity* generalizes *A General Theory of Optimal Algorithms* by J. F. Traub and H. Woźniakowski, Academic Press, New York, 1980, which was recently very ably reviewed by Ed Packel (this Review, 28 (1986), pp. 435–437). In the previous volume uncertainty was measured by a norm on a linear space. In this volume it is measured by nested families of subsets of a given space depending on a nonnegative real parameter and with nonempty intersection, so that any problem considered has a solution. In this fashion discrete and continuous problems can be treated in the same context. Despite the more general context, this book is easier to get into because it is less technical and has worked-out examples on binary search, integration, normed linear spaces, polynomial zeros, uniform approximation and division. There is also a chapter of applications to algebraic coding theory, distributed computation, continuous binary search, bin packing, nonlinear equations, linear equations, database security, Boolean functions, information theory and decision theory.

The problem addressed by this book is whether the problems studied can be solved up to preassigned uncertainty, and if they can be, how difficult is this to do? If algorithm is construed so broadly that any function mapping problems to solutions is considered an algorithm, then the Axiom of Choice and the hypothesis that the solution set is nonempty guarantee that there is always an algorithm to solve any problem. In this formulation the existence of an algorithm to solve a class of problems, approximately or exactly, is only interesting when the notion of algorithm is restricted or the information about the exact problem we are to solve is incomplete, only partially used or contaminated.

The first three chapters of this book fix the information and study the intrinsic error of any algorithm using that information. The arguments are mostly adversary arguments. The fourth chapter turns to optimal information and the issue of adaption. The fifth studies cost and is called complexity. The sixth chapter contains the applications. Finally, there are Appendices A–H. Worst-case analysis is used in this volume; an average case analysis is promised in a future volume.

There are various objections to the practicality of optimal algorithms. For example, if in integrating functions defined on the unit interval using functional evaluations at $n$ points we find that spacing these points evenly is optimal, then the transition from $n$ to $n+1$ points is awkward. Frequently we may be unsure as to which space our function shall be considered to lie in, so we do not know which worst-case analysis to apply. Special algorithms tailored to special spaces may be complex and we may prefer a simple algorithm which works well for a large class of problems and spaces. But in any case, having bounds in terms of optimal algorithms shows what is theoretically possible for comparison's sake, even if the resulting algorithms are for one reason or another impractical. Adversary arguments are clearly very powerful for establishing lower bounds, and it is in the formalization of the basic principles of these arguments in terms of the radius of information in the first three chapters of *Information, Uncertainty, Complexity* that I find the book at its strongest.

As a beginner browsing through the book, I find all of the arguments mathematically correct and the authors clear about their hypotheses and the limitations of the theory they propose. Yet I am not always convinced of the applicability of the theory to the examples being studied on a practical level. Let me give some examples starting with one from Chapter 4 of *Information, Uncertainty, Complexity*. I am thinking of a number between 0 and 15. Guess my number. We are all familiar with the usual divide and conquer technique of asking is the number between 0 and 7? and then branching to is the number between 0 and 3? or is the number between 8 and 11? depending on whether the answer to the first question is yes or no, etc. The information we get is adaptive in the sense that the question we ask depends on the previous information we have received. Moreover, the domain of definition of our question can be considered to have changed; first it is a question about all numbers 0 to 15, then about only those numbers from 0 to 7 or from 8 to 15. After four questions we have found our number. Chapter 4 deals with the problem of whether nonadaptive information can solve the same problem as adaptive information, and if so whether it requires more information. Nonadaptive information may be computed in parallel. In this case there are also four questions which solve the problem nonadaptively. Here are four questions which work. Let $A = (0\ 2\ 4\ 6\ 8\ 10\ 12\ 14)$, let $B = (0\ 1\ 4\ 5\ 8\ 9\ 12\ 13)$, let $C = (0\ 1\ 2\ 3\ 8\ 9\ 10\ 11)$ and let $D = (0\ 1\ 2\ 3\ 4\ 5\ 6\ 7)$. Is the number in $A$? Is the number in $B$? Is the number in $C$? Is the number in $D$? These four questions amount to asking if the least significant bit is zero, if the next is zero, etc. Thus they give enough information to solve the problem. So nonadaptive information is as strong as adaptive for this problem. Yet it would seem, since the fourth question we have asked in the nonadaptive setting is the same as the first we have asked in the adaptive setting, and since the answer to the adaptive question reduces our problem size by half, that there should be some preference for the adaptive information. This might be wrong, but these considerations seem compelling even if they go beyond the intentions of the example. Similar phenomena happen in other examples as well. Optimal algorithms in the worst case may no longer be optimal when the domain of definition of the problem is adaptively reduced, or the cost of evaluating information operators on subsets may go down. There is much more about this with regard to the integration problem in the thesis of Feng Gao "Nonasymptotic Error of Numerical Integration—an Average Analysis," Mathematics Department, University of California, Berkeley, 1986.

The cost of the information becomes an issue again for me later with respect to some real number problems. Example 4.1 studies the problem of division allowing for the exact operations of addition, subtraction and multiplication performed at unit cost. The problem is to evaluate $1/(3-f)$ for $f$ in the closed interval $-1$ to 1. Here it seems reasonable to me to assign the same cost of computation independent of the point since the problem is well conditioned, $f$ staying well away from 3. Thus the complexity analysis of this example seems reasonable to me. On the other hand it does not seem reasonable to me to assign unit cost to the functional evaluations of all functions and points in order to locate a zero of a continuous function defined on the closed unit interval, as the complexity analysis of the bisection algorithm example of Chapter 6 does, since the question is most sensitive for values near zero, and especially for functions which are near zero on the whole interval. More interesting to me in the section on nonlinear equations is the theorem of Wasilkowski, which is referred to there, that no algorithm using a finite amount of linear information can produce epsilon approximations to the roots of all complex polynomials.

I have recently received a survey paper from H. Woźniakowski entitled "Information Based Complexity" to appear in the *Annual Review of Computer Science*, 1 (1986), which concentrates more on continuous problems and the normed linear space setting.

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*Homogenization and Effective Moduli of Materials and Media.* Edited by J. L. ERICKSEN, D. KINDERLEHRER, R. KOHN AND J.-L. LIONS. Springer-Verlag, New York, 1986. x + 263 pp. $22.50. ISBN 0-387-96306-5. IMA Volumes in Mathematics and Its Applications, Vol. 1.

To the uninitiated I note that homogenization is a theory developed by F. Murat and L. Tartar which places in a functional analytic framework the following basic problem of applied mathematics: Given a function $u(x, y)$, where $u$ is periodic in $y$, what is the behavior in $\varepsilon$ as $\varepsilon \rightarrow 0$ of the function $u(x, (x/\varepsilon))$? The function $u$ is regarded as the macroscopic variable and we wish to determine its behavior by averaging the microscopic variable $y$. In fact, homogenization treats much broader issues than this, e.g., the periodic structure is not necessary but the example provides the flavor of the issue. Canonical physical examples arise in determining either the homogenized thermal conductivity or homogenized elasticity tensor of a periodic mixture of two different materials. These homogenized quantities represent the *effective moduli* of the mixture. Furthermore, as one learns to compute effective moduli, one sees that one may try to optimize them, e.g., make heat dissipate as rapidly as possible or make the material as strong as possible by an optimal choice of mixing strategy, i.e., perform *an optimal structural design.*

The volume under review is a collection of papers by world experts in both homogenization and optimal structural design. The emphasis of all the papers is on applications and examples. The papers are of high quality, well written, and happily one need not be a specialist to gain insight from reading them.

I recommend the collection to anyone interested in seeing what is happining on the applied side of homogenization and optimal structural design. For the record, the following papers are included in the volume:

*Generalized Plate Models and Optimal Design*, Martin P. Bendsoe; *The Effective Dielectric Coefficient of a Composite Medium: Rigorous Bounds from Analytic Properties*, David J. Bergman; *Variational Bounds on Darcy's Constant*, James G. Berryman; *Micromodeling of Void Growth and Collapse*, M. M. Carroll; *On Bounding the Effective Conductivity of Anisotropic Composites*, Robert V. Kohn and Graeme W.